Table 4 First four eigenvalues for symmetric-antisymmetric vibration modes ($\lambda^2 = \omega a^2 \sqrt{\rho/D}$)

	$\phi = b/a$								
Mode	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0
1	8.875	7.945	7.288	6.768	6.354	6.027	5.765	5.554	5.372
2	23.68	18.43	15.71	14.24	13.37	12.78	12.31	11.84	11.32
3	36.68	31.92	26.57	22.02	18.78	16.50	14.92	13.85	13.17
4	48.04	39.80	35.38	33.02	31.14	27.77	24.31	21.63	19.55

Table 5 First four eigenvalues for symmetric-antisymmetric vibration modes ($\lambda^{*2} = \omega b^2 \sqrt{\rho/D}$)

	$\phi_1 = a/b$								
Mode	1.0	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0
1	8.875	6.552	5.355	4.697	4.310	4.066	3.901	3.781	3.690
2	23.68	19.44	15.65	12.76	10.74	9.335	8.328	7.589	7.034
3	36.68	27.62	23.79	21.44	19.02	16.58	14.51	12.87	11.57
4	48.04	37.71	31.41	26.94	24.00	22.05	20.46	18.70	16.94

very quickly, and 20 point supports and 15 terms are enough to guarantee accuracy up to 4 digits. Therefore, it was decided to set the number of point supports K_p equal to 20 and the number of terms in the expansion K equal to 15 for all of the computed eigenvalues.

All of the results presented here are computed with a value of Poisson's ratio equal to 0.333. The first four eigenvalues for fully symmetric and fully antisymmetric vibration modes are given in Tables 2 and 3, where the aspect ratio ϕ varies between 1 and 3. In Tables 4 and 5 corresponding eigenvalues are tabulated for the symmetric-antisymmetric vibration modes, where the aspect ratio is allowed to vary from 1/3 to 3 because of nonsymmetry of this family of modes.

It should be pointed out that some eigenvalues for the title plate can be established in advance. For instance, the first eigenvalue, 4.806, of fully symmetric modes of the square plate ($\phi = 1$) in Table 2 is equal to the first eigenvalue for fully symmetric modes of free square plates.[1] This is because there are naturally two nodal lines along the diagonals for the mode shape of the latter, independently of whether the line support exists. Accordingly, the third and the fourth eigenvalues of square plates in Table 2 coincide with the fourth and the sixth of the associated modes of fully free plates. Those eigenvalues are usually called *inactive support eigenvalues* since in those cases the line support has no effect on the frequencies and mode shapes.

Conclusion

The superposition method is a very efficient tool for plate vibration analysis. This technique, with some modification to include line support, has successfully solved the title problem. Convergence is found to be rapid and the accuracy of frequencies is high. It is obvious that the analytical technique can be utilized for plates where the line support is given any configuration on the surface. It is hoped that the eigenvalues presented here will be useful to engineers for design purposes and to researchers for checking their findings.

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Vibration and Buckling of Laminated Plates with a Cutout in Hygrothermal Environment

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Introduction

RECENTLY, fiber reinforced plastic composites have been receiving more attention from scientists, engineers, and designers due to their superior properties. They are finding increased application in aerospace and other industries. Since polymer resin is used as a binding material, they are susceptible to environmental effects. Moisture and temperature have a significant effect on the performance of laminated plates due to the introduction of residual stresses, in addition to degradation in their elastic properties. The presence of a cutout further influences the strength and stiffness.

Of late, there is a growing interest to investigate various aspects of composite materials in hygrothermal environment.[1] The effects of moisture and temperature on the free vibration and buckling of laminated plates without a cutout have been considered earlier by the authors.[2,3] Virtually, there is no literature concerning the vibration and buckling of laminated plates with a cutout in hygrothermal environment. Chang and Shio[4] studied thermal buckling of isotropic and composite plates with a hole. A closed-form solution is presented for thermal buckling analysis of an annular isotropic plate with a circular hole, and finite element analysis is used to analyze composite plates with a circular cutout. Numerical results are shown for antisymmetric angle-ply laminates. Reddy[5] investigated large-amplitude flexural vibrations of composite plates with a square cutout using a finite element method. Results are

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shown for linear and nonlinear frequencies. Rajamani and Prabhakaran[6] studied dynamic response of composite plates with a cutout for simply supported and clamped boundary conditions. Natural frequencies of specially and generally orthotropic square plates with a square cutout are presented by expressing the effect of a cutout as an external loading function on the plates. Buckling behavior of compression loaded symmetrically laminated angle-ply plates with a hole is investigated by Nemeth[7] using finite element analysis and experiments.

In the present study, the free vibration and buckling of laminated composite plates with a circular cutout subjected to moisture and temperature are investigated. The analysis is carried out by the finite element method using an eight-noded isoparametric element, which takes the transverse shear deformation and rotary inertia into account. The conventional finite element formulation is modified to include hygrothermal effects. The effect of cutout size on fundamental natural frequencies and critical loads are studied for square symmetric and antisymmetric laminates with simply supported and clamped boundary conditions in the presence of moisture and temperature. The details of governing equations and finite element formulation are given in Refs. 2 and 3.

Results and Discussion

The entire plate is modeled with 36 elements. The accuracy of the present finite element analysis is checked for free vibration and buckling of isotropic plates with a circular cutout. It is observed that the accuracy is good. Material properties of graphite/epoxy lamina are used, which are taken from Chap. 8 of Ref. 8.

Figures 1 and 2 show the effect of cutout size on the fundamental natural frequency of simply supported symmetric and antisymmetric laminates subjected to uniform moisture concentration 1.0% and also in the absence of moisture. In the case of cross-ply laminates with a side-to-thickness ratio a/t of 10, the fundamental frequency decreases (d/a is about 0.45) and then increases with an increase in cutout size (Fig. 1). This is also true for the previous laminates when a/t is 40 in the absence of moisture, whereas in the presence of 1.0% uniform moisture concentration, there is a greater decrease in the fundamental frequency up to d/a of about 0.375 and then it increases sharply. Figure 2 shows that the fundamental frequency of angle-ply laminates increases with increase in cutout size, and the increase is sharp when d/a is about 0.2. Again, there is a greater increase in the fundamental frequency of angle-ply laminates when a/t is 40, subjected to uniform moisture concentration 1.0%.

In the case of clamped symmetric and antisymmetric laminates, the fundamental frequency increases with increase in cutout size; the increase is more when the laminates when a/t

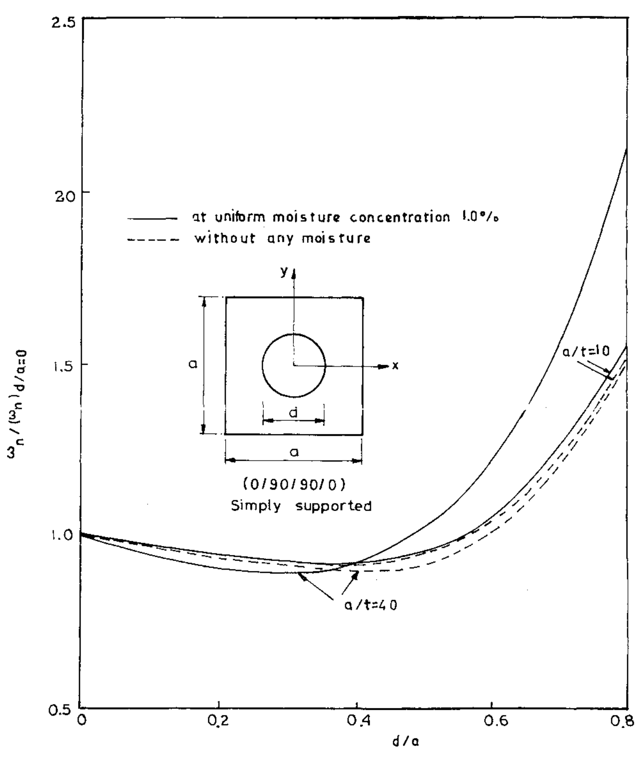

Fig. 1 Effect of cutout size on the fundamental natural frequency (symmetric cross ply).

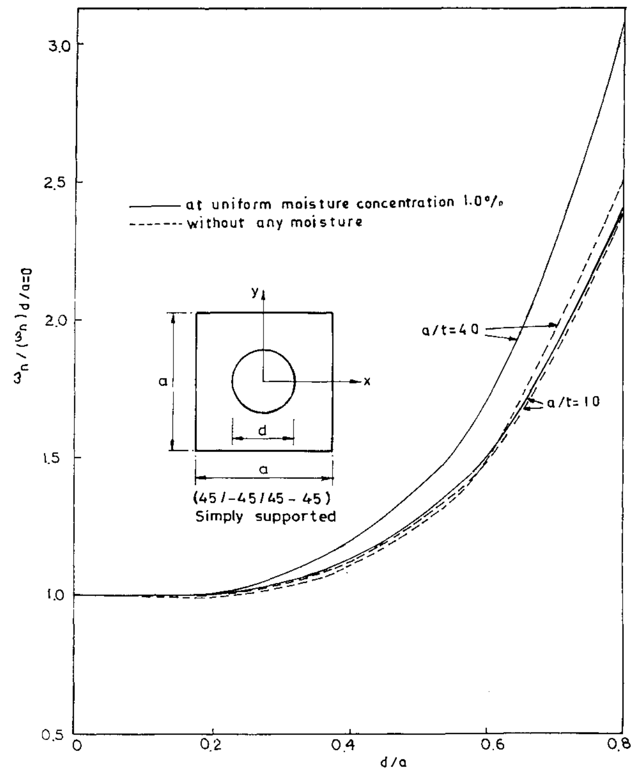

Fig. 2 Effect of cutout size on the fundamental natural frequency (antisymmetric angle ply).

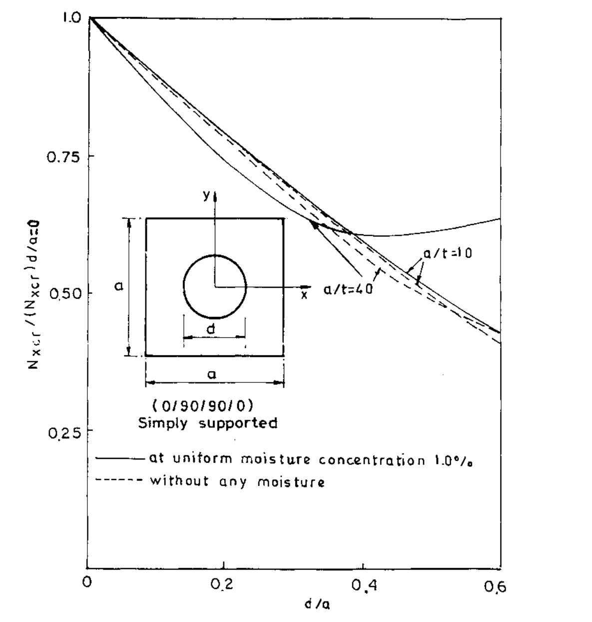

Fig. 3 Effect of cutout size on the critical load (symmetric cross ply).

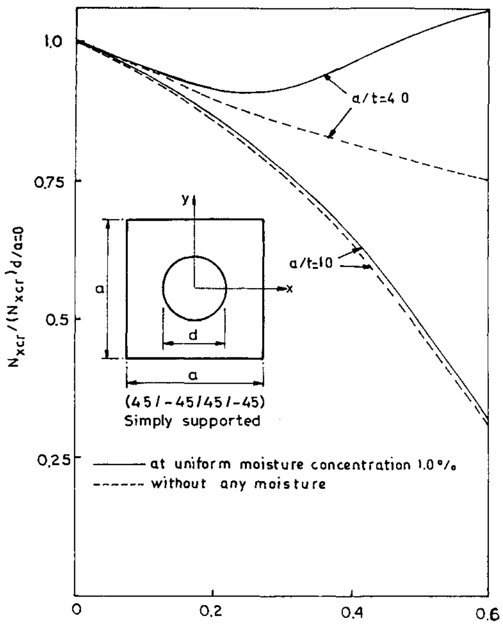

Fig. 4 Effect of cutout size on the critical load (antisymmetric angle ply).

of 40 are subjected to uniform moisture concentration of 1.0%. However, compared to the simply supported case, moisture presence has less effect on the fundamental frequency of clamped laminates with a cutout when a/t is 40.

The effect of cutout size on the critical loads of simply supported symmetric and antisymmetric laminates, subjected to 1.0% uniform moisture concentration and in the absence of moisture, is shown in Figs. 3 and 4. In the case of laminates when a/t is 10, the critical loads decrease with increase in cutout size. The critical loads of the laminates when a/t is 40 also decrease with increase in cutout size in the absence of moisture; whereas, in the presence of 1.0% uniform moisture concentration, they first decrease (d/a is about 0.4 for cross-ply laminates, d/a are about 0.3 and 0.25 in respect of angle-ply symmetric and antisymmetric laminates, respectively) and then increase.

The critical loads of clamped symmetric and antisymmetric laminates when a/t is 10 are reduced with increase in cutout size, but the initial reduction is sharp for cross-ply laminates. The critical loads of the clamped laminates when a/t is 40 are also reduced with increase in cutout size in the absence of moisture; whereas in the presence of moisture, the critical loads decrease and/or increase as the case may be.

Conclusions

The increase in the fundamental natural frequencies of thin laminates, with increase in cutout size, is greater in the presence of moisture. Moisture presence has negligible effect on the increase in the fundamental natural frequencies of thick laminates with increase in cutout size. The buckling behavior of thick laminates with a cutout is almost unchanged due to the presence of moisture. The critical loads of thin laminates with a cutout are very much affected in the presence of moisture.

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Torsional Stiffness for Circular Orthotropic Beams

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Nomenclature

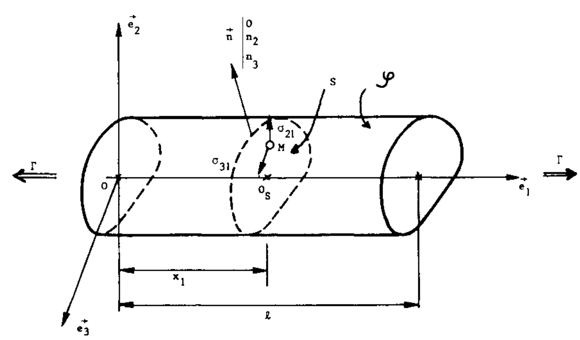
A	= tensor
$A{:}B$	= double dot product: $A_{ij}B_{ji}$
$A\cdot u$	= simple dot product: $A_{ji}u_ie_j$
u	= vector
$u\cdot v$	= simple dot product (inner product): u_iv_i
∇	= gradient operator: $\varphi\nabla = \varphi_{,i}e_i$
$\cdot\nabla$	= divergence operator: $u\cdot\nabla = u_{i,i}$, $A\cdot\nabla = A_{ij,j}e_i$
$\wedge$	= vector product or cross product

Introduction

FOR a long time, solutions for torsion of beam structures have been studied and solutions for many problems can be found in the literature; see Refs. 1-5, for example. Warping functions are known analytically for some problems, and either may be obtained using computers. But, in most of the cases the medium is isotropic. In this Note, we pose the problem for an orthotropic medium, and we give the solution for a circular section and propose some evident applications.

Development

We consider a circular cylinder with axes in one of the directions of orthotropy of the medium, and we complete the basis by the two other directions of orthotropy. The body in equilibrium is only acted upon by a couple Γe_1 on its base $x_1 = \ell$, and $-\Gamma e_1$ on its base $x_1 = 0$.



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